TOPIC 8-7: Parallel Lines & Proportional Parts

Proportions can be used to find the lengths of segments determined by parallel lines.

**Triangle Proportionality Theorem:** If a line is parallel to one side of a triangle and intersects the other two sides in two distinct points, then it separates these sides into segments of proportional length.

\[
\frac{CB}{CA} = \frac{CD}{CA}; \quad \frac{CB}{BA} = \frac{DE}{CA} \]

**Example 1** True or False?

(a) \( \frac{FA}{HA} = \frac{FB}{TB} \)  
(b) \( \frac{FT}{FA} = \frac{FB}{TB} \)  
(c) \( \frac{FH}{FT} = \frac{HA}{TB} \)  
(d) \( \frac{FA}{FH} = \frac{FT}{TB} \)

**Example 2** Find the value of ‘x’.

\[
\frac{FA}{HA} = \frac{FB}{TB} \]
EXAMPLE 3  Find the values of ‘x’.

Likewise, proportional parts of a triangle can be used to prove the converse of this theorem.

THEOREM: If a line intersects two sides of a triangle and separates the sides into corresponding segments of proportional lengths, then the line is parallel to the third side.

EXAMPLE 4  In \(\triangle EFG\), \(EG = 15\), \(EH = 5\), and \(LG\) is twice \(FL\).
Determine whether \(HL \parallel EF\).

EXAMPLE 5  In \(\triangle ABC\), find ‘x’ so that \(DE \parallel CB\).

\[
\begin{align*}
AC &= 30 \\
AD &= 10 \\
AE &= 22 \\
EB &= x + 4
\end{align*}
\]
**THEOREM:** A segment whose endpoints are the midpoints of two sides of a triangle is parallel to the third side of the triangle, and its length is half the length of the third side.

**EXAMPLE 6** Find the values of ‘x’ and ‘y’.

![Diagram](image1)

**THEOREM:** If three or more parallel lines intersect two transversals, then they cut off the transversals proportionally.

\[
\frac{PQ}{QR} = \frac{DE}{OR} \quad \text{or} \quad \frac{PQ}{DE} = \frac{QR}{OR}
\]

**EXAMPLE 7** TRUE or FALSE?

(a) \( \frac{a}{b} = \frac{c}{d} \)  
(b) \( \frac{a}{c} = \frac{b}{d} \)  
(c) \( \frac{a}{d} = \frac{c}{b} \)  
(d) \( \frac{b}{c} = \frac{a}{d} \)
EXAMPLE 8  Find the value of ‘x’.

\[
\begin{array}{c}
5 \\
7 \\
x \\
9
\end{array}
\]

EXAMPLE 9  Find the values of ‘x’ and ‘y’.

\[
\begin{array}{c}
y \\
x + 3 \\
3 \\
4 \\
2y - 5
\end{array}
\]

THEOREM: If three or more parallel lines cut off congruent segments on one transversal, then they cut off congruent segment on every transversal.

EXAMPLE 10  Find the value of ‘x’.

\[
\begin{array}{c}
x + 1 \\
x + 3 \\
2x - 5
\end{array}
\]